The Islamia University Of Bahawalpur,
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Course: Numerical Analysis **Program:** BSCS-V (Spring 2020)

Topic: Initial Value Problem

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Initial Value problems for ordinary D.Eq's.

Sometimes we need to find the solutions of D-Eq's. subject to a supplementary conditions. Two types of conditions will be often encountered.

i) → Initial value conditions (I.V.C)
ii) → Boundary value conditions (B.V.C)

⇒ I.V.C If the conditions relate to one value of independent variable such as $y = y_0$ at $x = x_0$

written as $y(x_0) = y_0$

and $\frac{dy}{dx} = y'(x_0)$ at $x = x_0$

It is also called one-point boundary conditions

Here x_0 is called the initial point.

⇒ Initial Value problems:- It's a process finding solution of Differential Eq. using I.V.C

An I.V.P consist of

(i) D.Eq. (Any order)

(ii) *I.V.C's*

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(iii) Solution of Eq (Sometimes)

For exp:

$$\frac{dy}{dx} = 2x \quad \text{--- (1)}$$

Such that $y(1) = 4$ | $x_0 = 1$, $y_0 = 4$

& General sol. $y = x^2 + c$

This is the I.V.P

Sol/ Since General solution of eq (1)

$$y = x^2 + c$$

c — arbitrary constt.

$\frac{dy}{dx} = 2x$

$dy = 2x\,dx$

Integrating b-s

$\int dy = 2\int x\,dx$

$y = x^2 + c$

As $y(1) = 4$

i-e $x = 1$

$$y(1) = 1^2 + c$$

$$4 = 1 + c$$

$$\Rightarrow c = 4 - 1$$

$$\boxed{c = 3}$$

Thus

$$y = x^2 + 3$$

is the solution of this I.V.P

Assignment
d – Exp

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$$\frac{d^2y}{dx^2} + y = 0$$

I.V.C

→ $y(1) = 3$

→ $y'(1) = -4$

G. Sol. $y = A\sin x + B\cos x$

Solution

$$\frac{d^2y}{dx^2} + y = 0 \quad — 1$$

I.V.C.

$x = 1$

$y(1) = 3$

$y'(1) = -4$

Since General solution.

$$y = A\sin x + B\cos x \quad — 2$$

$\frac{dy}{dx}$ $y' = A\cos x - B\sin x$

Two Arbitrary Constt. i.e order-2

For $x = 1$ & $y(1) = 3$

$y(1) = A\sin(1) + B\cos(1)$

$3 = A\sin(1) + B\cos(1) \quad — 3$

For $x = 1$ & $y'(1) = -4$

$\cancel{-4 = A\cos(1) + B\sin(1)}$

$-4 = A\cos(1) - B\sin 1 \quad — 4$

Solve eq 3 & 4 for finding 'A' & 'B'

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$$A = -4\cos 1 + 3\sin 1$$
$$B = 3\cos 1 + 4\sin 1$$

For these values of 'A' & 'B' eq (2) is required solution of the I.V.P

3rd Exp:

$$\frac{dy}{dx} = -\frac{x}{y} \rightarrow (1) \quad y(3) = 4$$

G. Sol. $x^2 + y^2 = c^2$

Solution:

$$y^2 = c^2 - x^2$$
$$y = \sqrt{c^2 - x^2} \rightarrow (2)$$

For $x = 3$; $y(3) = 4$

$$(3)^2 + (4)^2 = c^2$$
$$9 + 16 = c^2$$
$$25 = c^2$$
$$\Rightarrow \boxed{c = 5}$$

$y\,dy = -x\,dx$
$\int y\,dy = -\int x\,dx$
$\frac{y^2}{2} = -\frac{x^2}{2} + c'$
$y^2 = -x^2 + 2c'$
$x^2 + y^2 = 2c'$
Let $2c' = c^2$
$x^2 + y^2 = c^2$
Eq. of circle with centre at origin.

Now from eq (2)

$y = \sqrt{25 - x^2}$ is required particular sol. of eq (1)

Example : Solve the initial value problem?

2nd Method

$$\frac{dy}{dx} = \frac{3x^2+4x+2}{2(y-1)} \qquad y(0) = -1$$

$$\int 2(y-1)\,dy = \int (3x^2+4x+2)\,dx$$

$$2\int y\,dy - 2\int 1\,dy = 3\int x^2\,dx + 4\int x\,dx + 2\int dx$$

$$2\frac{y^2}{2} - 2y = x^3 + 2x^2 + 2x + c_1$$

$$y^2 - 2y = x^3 + 2x^2 + 2x + C_1$$

$$y^2 - 2y + (1)^2 = x^3 + 2x^2 + 2x + c_1 + 1$$

$$(y-1)^2 = x^3 + 2x^2 + 2x + c_1 + 1 \qquad y - 1 = \sqrt{x^3 + 2x^2 + 2x + c_1 + 1}$$

$$y - 1 = \sqrt{x^3 + 2x^2 + 2x + c_2}$$

$$y = \sqrt{x^3 + 2x^2 + 2x + c_2} + 1$$

$$c_2 = c_1 + 1$$

Apply I.V.C

$$-1 = \sqrt{(0)^3 + 2(0)^2 + 2(0) + c_2} + 1$$

$$-1 - 1 = \sqrt{c_2}$$

$$-2 = \sqrt{c_2}$$

i.e $C_2 = 4$

$$Y = \sqrt{x^8 + 2x^2 + 2x + 4} + 1$$

Assignment No 1

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Assignment:- (I.V.P)

Q1

$$x\frac{dy}{dx} + 2y = 4x^2$$

For $y(1) = 2$

G.S $y = x^2 + \frac{c}{x^2}$

Q2

$$x^3 \frac{d^3y}{dx^3} - 3x^2\frac{d^2y}{dx^2} + 6x\frac{dy}{dx} - 6y = 0$$

For $y(2) = 0$; $y'(2) = 2$, $y''(2) = 6$

G.S $y = c_1 x + c_2 x^2 + c_3 x^3$

Q3

$$\frac{d^2y}{dx^2} + y = 0$$

For $y(1) = 3$
$y'(1) = -4$

G.Sol. $y = A\sin x + B\cos x$